Lazzarini's little sticks

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Abstract

In 1910, Mario Lazzarini claimed to have approximated the value of pi to an unbelievable six decimal places with an ingenious automaton. Even today, mathematicians are still indignant: Lazzarini must have been a fraud. More likely, though, he was a prankster.

An 'enormity', the editor of *Nature* called it, a tale that 'should be a warning to all those who pollute the literature that their misdeeds will follow them to the grave.[1] Clearly, John Maddox was upset: he had just read an article published four months earlier by the American mathematician Lee Badger.[2] Badger had delved into a paper published more than ninety years earlier by an Italian colleague and shown that its results could not possibly be *bona fide.* That Italian colleague was Mario Lazzarini, and his article dealt with an experimental determination of the number pi — the number we know best from mathematical formulas to calculate the circumference or the area of a circle from its diameter.[3] Maddox got himself rather worked up: 'Badger describes the result as "lucky". That is a charitable way of putting it. The truth is that if Lazzarini's result had been published in 1994 and not 1901, it would be called a barefaced fraud.'

If only Maddox had read Lazzarini's original paper instead of Badger's — he would have gotten red-faced from laughter, not from indignation.

Parquet floor

The idea of approaching pi not through mathematics, but through experiment, has a long history. The eighteenth century was the century in which mathematicians discovered probability — the calculation of games of luck and gambling. It provided countless solutions to problems that had previously seemed unsolvable, and of course countless new problems to be solved. One of these was a game called *franc-carreaux:* players throw a coin on a tiled floor: one player

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1. J. Maddox: False calculation of π by experiment. *Nature* vol. 370, no. 6488 (4 August 1994) p. 323.

2. L. Badger: Lazzarini's lucky approximation of π. *Mathematics Magazine* vol. 67, no. 2 (April 1994), p. 83–91.

3. Mario Lazzarini: Un'applicazione del calcolo dell probilità alla ricerca sperimentale di un valore approsimato di π. *Periodico di matematica per l'insegnamento secondario,* vol. 17, no. 3 (November–December 1901) p. 140–143.

bets the coin will land wholly within a tile, the other that the coin will touch one or more joints. The chances depend, of course, on the relative sizes of the tile and the coin, and in 1777 the French scholar George-Louis Leclerc, Count of Buffon, 'amused himself' by calculating what this ratio should be to give the players even odds. Next, he posed a more difficult problem: not with a coin and tiles, but with a stick and lines:

> Je suppose que, dans une chambre dont le parquet est simplement divisé par des points parallèles, on jette en l'air une baguette, et que l'un des joueurs parie que la baguette ne croisera aucune des parallèles du parquet, et que l'autre au contraire parie que la baguette croisera quelques-unes de ces parallèles ; on demande le sort de ces deux joueurs.[4]

He added that 'one can play this game on a checkerboard with a sewing needle or a headless pin'.

With some simple higher mathematics, Buffon could prove that the odds for the two players are equal if the length of the stick is about three-fourths the width of the boards. For good measure, he also calculated the fate of the players for a stick on a differently tiled floors. Note, however, there is no mention of pi in any of his calculations.

Only about fifty years later, in 1812, his compatriot Pierre-Simon Marquis de Laplace picked up the idea, and he was able to calculate the absolute probability that 'a very thin cylinder' would hit a line. Again, this depends on both the length of the cylinder and the distance between the lines. Laplace called the length of the cylinder $2r$ and the distance a (and by then mathematicians used the Greek letter π for 'pi'):

> Si l'on projette un grand nombre de fois ce cylindre, le rapport du nombre de fois où le cylindre encontrera l'une des divisions du plan, au nombre total des projections, sera à très-peu près, la valeur de $4r/\pi$, ce qui fera connaître la valeur de la circonférence 2π.[5]

This not only provides a solution to the problem, but also a nice way to estimate pi by experiment: if we know the length of the stick and the distance between the lines, then, after a lot of tosses, pi rolls out. This can also be done with a sewing needle or a headless pin, and the experiment, although Buffon did not propose it at all, has become known as 'Buffon's needle': throw a needle on a lined surface, keep track of the number of hits H and the total number of throws N, and calculate pi following Laplace's formula (now l is the length of the needle, a is again the distance between the lines):

$$\pi = (2l/a) \times (N/H)$$

So if, for example, the needle is 25 millimeters long and the distance between the lines is 26 millimeters, after 100 throws and 60 hits we get an estimate of pi of 3.205,128 (that many decimal places is nonsensical, of course — more on that later).

4. *Essai d'arithmétique morale*, 1777, par. XXIII.
5. *Théorie analytique des probabilités*, 1812, chap. V.

Trial and error

There really have been people who have tried to approximate π in this way. If they had calculated in advance how many attempts they would have to make to determine even the first decimal place with reasonable certainty, they would have looked for something else to do, but the tenacity of mathematicians should never be underestimated.

We will not name all the heroes — from the popular science books we take R. Ambrose Smith from Aberdeen who in 1855 after 3204 tosses arrived at a pi of 3.1412;[6] an unnamed student of Augustus de Morgan with 600 throws and 3.137;[7] and one American captain O. C. Fox who in 1864, according to his friend the American astronomer Asaph Hall, came up with three values, the best of which was 3.1416 after exactly 939 throws.[8] The latter attempt is indeed remarkable, given pi's true value:

$$\pi = 3.141{,}592{,}653{,}589{,}793{,}238{,}...$$

However, the gold medal for the most accurate approximation surely has to go to our Italian protagonist, Mario Lazzarini. He reported, with 1808 hits in 3408 tosses, a stunningly accurate approximation: 3.141,592,9 — correct to six decimal places. That was the enormity on which John Maddox nearly suffocated.

It also looks, *a prima vista,* far too good to be true, and of course Mario Lazzarini's results had been frowned upon before Lee Badger and John Maddox got involved. Rather, it is remarkable that many mathematicians took his finding at face value and really considered it a most amusing proof of mathematical and experimental ingenuity. Again, we won't mention everyone, but we find names like George Gamow,[9] Jacob Bronowski[10] and Edward Kasner and James Newman — 'One could scarcely expect to find a better example of the inter-relatedness of all mathematics,' they wrote in their best-selling *Mathematics and the imagination.*[11]

The first to raise suspicions against Lazzarini seems to have been the American mathematician Julian Coolidge, in 1925.[12] He suspected Lazzarini was 'watching his step' to reach his spectacular result.

Things heated up in the sixties. The Canadian mathematician Norman Gridgeman cast his doubts in *Scripta Mathematica,*[13] and a year later, Thomas O'Beirne devoted his famous column 'Puzzles and paradoxes' in *New Scientist* to the issue — without knowing of Gridgeman's article.[14]

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6. A. de Morgan: Quadrature of the circle. *Cyclopædia of Arts and Sciences,* vol. VI. London: 1861, p. 868–874.
7. A. de Morgan: *A budget of paradoxes.* London: 1872, p. 171–172.
8. A. Hall: On an experimental determination of π. *The messenger of mathematics,* vol. II (1873) p. 113–114.
9. G. Gamow: *One two three ... infinity. Facts and speculations of science.* New York: 1947, p. 222.
10. J. Bronowski: *The common sense of science.* London: 1951, p. 86.
11. E. Kasner and J. Newman: *Mathematics and the imagination.* New York: 1945, p 214–215.
12. J. L. Coolidge: *An introduction to mathematical probability.* Oxford: 1925, p. 81–82.
13. N. T. Gridgeman: Geometric probability and the number π. *Scripta Mathematica,* vol. 25, no. 3 (1960), p. 183–195.
14. T. H. O'Beirne: Puzzles and paradoxes 23: How to π with statistics. *New Scientist,* vol. 10, no. 238 (8 June 1961), p. 598.

Then, in 1994, Lee Badger delivered the final blow with heavy mathematical artillery.

Highschool

Mario Lazzarini published his article in the *Periodico di matematica per l'insegnamento secondario.* This is, as the name suggests, not a mathematical journal, but a journal for mathematics teachers. This matters for the remainder of the story; in all articles on Lazzarini, the journal is invariably called *Periodico di matematica* as if it were an authoritative scientific journal. (Also, one can see mistakes in citations being propagated, but that's another matter.)

Hence, whether Lazzarini was really an 'Italian mathematician', as he is always claimed to have been, is also questionable: it is more likely that he was a mathematics teacher — according to another article of his in the *Periodico,* he lived andor worked in Massa, in Tuscany.

A search for his further antecedents, undertaken for me by Ilaria Vercillo of the Biblioteca Casanatense in Rome, failed to bring up further details. Lazzarini seems to have published no articles beyond a few in the *Periodico* and a chapter in a book on Fibonacci; he seems to have faded into the fog of the history of mathematics.

Washing machine

In this fog, however, Lazzarini's entire experiment has also disappeared — and that's a shame, because it gives his critics all the more opportunity to cast him in a bad light. Gridgeman had his opinion ready before he had even read Lazzarini's article and only tells half the story, O'Beirne and Badger also don't bother to explain exactly how Lazzarini's experiments worked.

However, in his article in the *Periodico,* Lazzarini goes to quite great lengths to describe his device — not for nothing.

He has, he says, built an ingenious machine that had completely automated the falling and measuring process. Inside a horizontally rotating drum, diameter 17 centimeter, there are two barriers on opposite sides, parallel to the axis of the drum (a bit like in some modern washing machines). A stick lying at the bottom of the drum is taken up by a barrier and comes down at its highest point. In the middle of the drum is a grid of parallel wires that the stick, falling down, may or may not hit. If the stick lands on a wire, the grid goes down slightly, and because it is mounted on a long movable shaft, the other end bounces up. On that other end is a writing stylus that presses down on a rolling strip of telegraph paper. If the grid goes down, the stylus goes up and the line is interrupted. The stick lands, and is taken up again by one of the barriers. The number of rotations of the drum are registered autmatically as well, so all the experimenter has to do is count the number of interruptions in the line on the paper, fill in the values in the formula, and find his pi.

Results

And here are the results Lazzarini obtained with this method — 'Ed ecco i risultati da me ottenuti con questo mezzo'. First, he says, he had placed the wires on the grid parallel to the axis of the cylinder. That did not work very well: the distance between the wires was 2.6 centimeters, and with 100 attempts he had 60 hits, with 500 attempts 276. Since his stick was 2.5 centimeters long, this gives estimates for pi of 3.205 and 3.484 (Lazzarini gives 3.483; it seems he rather truncates than rounds).

N	H	pi
100	60	3.205
500	276	3.483

Apparently, this was not to his liking, so he turned his grid one quarter turn. Now things went better. Lazzarini reports, in two tables, two series of measurements: the first one again with a distance between the threads of 2.6 centimeters:

N	H	pi
100	62	3.101
200	122	3.152
1000	611	3.147
2000	1229	3.126
3000	1840	3.135
4000	2448	3.142

Finally, the results of the second series, where he increased the gaps to 3 centimeters:

N	H	pi
100	53	3.144
200	107	3.115
1000	524	3.180
2000	1060	3.1446
3000	1591	3.142
3408	1808	3.1415929
4000	2122	3.1416

Indeed, the score at $N = 3408$ stands out. Not only because 3408 is not a round number, but also because pi is suddenly calculated to seven decimal places, while the others only to three or four. And the first six of these seven decimal places are correct.

Seriously?

Surely, it is inconceivable that any of Lazzarini's colleagues took this result serious. Literally everything indicates a joke, a joke his readers might want to use in their mathematics classes. Especially if their students have just learned the miraculous approximation of pi from the Chinese mathematician Zu Chongzhi, 355/113, in the fifth century. Because, obviously, this is how Lazzarini's fraction works:

$$(5/3) \times (3408/1808) = 355/113$$

and in 5/3 we recognize $2l/a$, twice the length of Lazzarini's stick divided by the space between the threads in his third series.

It must have been equally clear, even to mathematics teachers at the beginning of the twentieth century, that this precision doesn't make any sense at all: if numerator and denominator have four digits, the decimal fraction should not have seven decimal places. Not to mention measurement uncertainties, the possibility of counting errors, and all other kinds of experimental hiccups.

And all students, not yet being mathematicians, will probably protest right away that the laws of gravity forbid that a stick ever goes all the way up to the top of the drum: after a quarter turn it will just roll off the barrier back to the bottom — no stick will even hit the grid.

So there's really no need for mathematics or probability here: all the numbers have been made up.

But because all of his commentators focus on the numbers and omit the details of Lazzarini's presentation and the journal's target audience, it seems as if our Italian committed a gross scientific transgression. Most critical authors just mention the isolated fraction 3408/1808 and immediately draw a red card. Gridgeman is the only author trying to describe Lazzarini's device. He certainly smells a rat, but he does not seem to have seen through Lazzarini's description. He goes as far as: 'Whether this machine ever existed outside the pages of the journal is a titilating question.'

Badger gives Lazzarini's intermediate estimates, and finds in them all the more evidence of fraud: these results are also too good to be true, and together they are completely inconceivable: 'With the normal approximation to the binomial distribution, the probability is less than 0.00003'. This is using a sledgehammer to crack a nut — if only there was a nut to crack.

Fraud and fun

Of course, there is no way to prove that Lazzarini was just having some pedagogical fun, but this explanation seems a lot more plausible than that of malicious intent.

To be sure, it is not always easy to tell the difference between a scientific fraud and a joke. Thanks to the Christmas issues of the medical journal *BMJ*, we know that praying can cure blood poisoning[15] and that the 'man flu' really does exist,[16] but sometimes other journals, even the lofty *New England Journal of Medicine,* like to lead their readers astray with an article meant (or not meant) to be funny, too.[17]

Whether all this droll science doesn't just lead to more confusion is another question.[18] Maybe poor Mario Lazzarini has an answer.

15. L. Leibovici: Effects of remote, retroactive intercessory prayer on outcomes in patients with bloodstream infection: randomised controlled trial. *BMJ* vol. 323, no. 7327 (21 December 2001), p. 1450–1451.
16. K. Sue: The science behind 'man flu'. *BMJ* vol. 359, no. 8134 (11 December 2017), j5560.
17. F. H. Messerli: Chocolate consumption, cognitive function, and Nobel laureates. *New England Journal of Medicine,* vol. 367, no. 16 (18 October 2012), p. 1562–1564.
18. M. Ronagh, L. Souder: The ethics of ironic science in its search for spoof. *Science and Engineering ethics,* vol 21, no. 6 (December 2015), p. 1537–1549.